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Possible experiment on quantum Bayes theorem

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Abstract. We propose a solid-state experiment to study the process of continuous quantum measurement of a qubit state. The experiment would verify that the evolution of a qubit during the measurement is governed by the information obtained from the detector (Quantum Bayes Theorem). In particular, it can show that an individual qubit remains coherent during the measurement, in contrast to decoherence for the ensemble of qubits. The experiment can be carried out using quantum dots, single-electron transistors, or SQUIDs.

The problem of quantum measurement (wavefunction collapse) remains controversial for over seventy years. The interest to the problem is renewed nowadays because of its direct relation to quantum computing [1] and also because the progress in experimental techniques makes it now possible to resolve some of the controversial issues, which were discussed earlier only from the philosophical or mathematical points of view. One of such issues is the continuous measurement of a quantum system.

The success of the theory describing interaction of a quantum object with environment [2] has lead to an opinion common nowadays that the collapse postulate is a needless part of the quantum mechanics and can be derived from the Schrödinger equation tracing out the detector degrees of freedom. Then the measurement process is described by gradual decoherence of the measured object. Since the procedure requires ensemble averaging, the "modern philosophy" of quantum mechanics says that only ensemble-averaged quantities make sense while the discussion of the evolution of an individual quantum system is meaningless. This claim sharply contradicts, however, the point of view of "old" textbooks. Moreover, the modern approach cannot actually reproduce the "orthodox" collapse postulate, but only its ensemble-averaged version. The controversy has acquired a practical aspect since in the proposed solid-state quantum computers the qubit measurement is necessarily continuous (not instantaneous).

Recently developed Bayesian formalism [3] (for earlier somewhat similar theories see, e.g. [4]) reconciles the modern and orthodox approaches and allows us to describe the continuous measurement of an *individual* qubit. It shows that if the detector is good enough (basically, its sensitivity should be quantum-limited), then the qubit remains coherent (in a pure state) during the process of measurement, while the gradual decoherence claimed by the modern approach is just a result of averaging over the ensemble of measurement results. The Bayesian formalism can be applied for the realization of continuous quantum feedback control of a qubit and qubit purification, that is impossible using the ensemble-averaged formalism.

Since the issue remains controversial, it is important to show experimentally that the individual qubit stays coherent during measurement, in contrast to the claim of the modern approach. This paper describes such an experiment which can be realized at the present-day level of technology (for discussion of more direct experiments which, however, are too difficult for realization, see Ref. [3]).

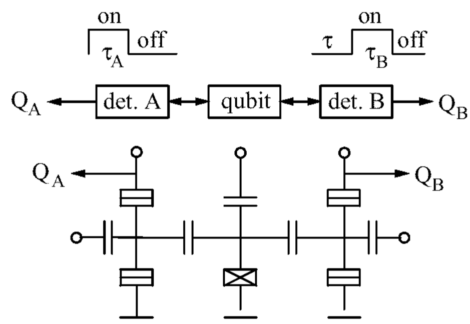

Fig. 1. Schematic of two-detector correlation experiment using Cooper-pair box and two single-electron transistors.

Within the Bayesian formalism the evolution of the density matrix ρ of the measured qubit (two-level system with the tunneling strength H and energy asymmetry ε) is described by equations

$$\dot{\rho}_{11} = -2H\mathrm{Im}\rho_{12} + (2\Delta I/S)\,\rho_{11}\rho_{22}[I(t) - I_0], \tag{1}$$

$$\dot{\rho}_{12} = \imath\varepsilon\rho_{12} + \imath H(\rho_{11} - \rho_{22}) - (\Delta I/S)\,(\rho_{11} - \rho_{22})[I(t) - I_0]\,\rho_{12} - \gamma\,\rho_{12}, \tag{2}$$

where $I(t)$ is the continuous detector output (we assume current), $\Delta I = I_1 - I_2$ is the difference between two average currents corresponding to qubit states $|1\rangle$ and $|2\rangle$, $I_0 \equiv (I_1 + I_2)/2 \gg |\Delta I|$, S is the output detector noise, and γ is the decoherence rate due to detector nonideality (for example, $\gamma = 0$ for quantum point contact), while the ensemble-averaged decoherence is $\Gamma = \gamma + (\Delta I)^2/4S$ (the detector ideality can be characterized by the parameter $\eta \equiv 1 - \gamma/\Gamma$).

The idea of the experiment is to use two detectors (A and B) connected to the same qubit (Fig. 1). The detectors are switched on for short periods of time by two shifted in time voltage pulses (one for each detector) with durations τ_A and τ_B, supplied from the outside. The output signal from the detector A is the total charge $Q_A = \int_0^{\tau_A} I_A(t)\,dt$ passed during the measurement period. Similarly, the output from the detector B is $Q_B = \int_\tau^{\tau+\tau_B} I_B(t)\,dt$, where τ is the time shift between pulses. If the measurement by the detector A changes the qubit density matrix, it will affect the result of measurement B. Repeating the experiment many times (with the same initial qubit state) we can obtain the probability distribution $P(Q_A, Q_B|\tau)$ of different outcomes, which contains the information about the effect of the quantum measurement on the qubit density matrix.

Figure 1 shows the realization of the experiment using single-electron transistors as detectors. Qubit is realized by the Cooper-pair box so that the electric charge of the central island can be in coherent combination of two discrete charge states. Another similar setup is two quantum point contacts measuring the charge state of a double-quantum-dot qubit. One more setup is the 3-SQUID experiment in which the qubit is realized by one SQUID while two other SQUIDs are in the detecting regime.

The ensemble-averaged formalism implies the absence of correlations between $\rho(t)$ and $I(t)$, so the average result of the second measurement $\overline{Q}_B(Q_A,\tau) \equiv \int Q_B\, P(Q_A, Q_B|\tau)\, dQ_B$ should not depend on Q_A. The Bayesian formalism (1)-(2) makes the different prediction: $\overline{Q}_B$ does depend on Q_A.

For simplicity let us assume symmetric qubit, $\varepsilon = 0$, which is initially in the ground state, $\rho_{11} = \rho_{22} = \rho_{12} = 0.5$, and also assume relatively strong coupling between the qubit and detectors, $(\Delta I_A)^2/HS_A \gg 1$, $(\Delta I_B)^2/HS_B \gg 1$ (subscripts A and B correspond to two detectors), so that we can neglect the qubit evolution due to finite H during the measurement periods τ_A and τ_B, which are assumed to be on the order of $S_{A,B}/(\Delta I_{A,B})^2$. Then from Eqs. (1)-(2) it follows that the first measurement only "partially" localizes the qubit state and after obtaining the result Q_A from the first measurement the qubit density matrix is

$$\rho_{11}(\tau_A) - \rho_{22}(\tau_A) = \tanh\left[\left[(Q_A - \tau_A I_{2A})^2 - (Q_A - \tau_A I_{1A})^2\right]/2S_A\tau_A\right], \quad (3)$$

$$\rho_{12}(\tau_A) = [\rho_{11}(\tau_A)\,\rho_{22}(\tau_A)]^{1/2}\exp(-\gamma_A\tau_A)\,. \quad (4)$$

Here Eq. (3) is the direct consequence of the Bayes formula, so this result can be called "Quantum Bayes Theorem" [5]. The qubit performs the free evolution during the time $\tau - \tau_A$ between measurements (here we neglect $\tau_A \ll \tau$) and the average result of the second measurement $\overline{Q}_B = \tau_B(I_{2B} + \rho_{11}(\tau)\Delta I_B)$ depends on Q_A in the following way (Fig. 2a):

$$\begin{aligned}\delta_B = (H/\Omega)\tanh&\left[\left[(Q_A - \tau_A I_{2A})^2 - (Q_A - \tau_A I_{1A})^2\right]/2S_A\tau_A\right]\\ &\times\cos[\Omega\tau - \arcsin(\gamma_f/4H)]\exp(-\gamma_f\tau/2)\,,\end{aligned} \quad (5)$$

where $\delta_B \equiv (\overline{Q}_B - \tau_B I_{0B})/\tau_B\Delta I_B$, γ_f is the dephasing with both detectors switched off, and $\Omega = (4H^2 - \gamma_f^2/4)^{1/2}$ is the frequency of quantum oscillations. Notice that δ_B changes sign together with the sign of $Q_A - \tau_A I_{0A}$, while the phase of oscillations is a piece-constant function of Q_A.

The dependence becomes quite different if the $\pi/2$ pulse is applied to the qubit immediately after the first measurement, that multiplies $\rho_{12}(\tau_A)$ given by Eq. (4) by the imaginary unit. In this case (Fig. 2b) $\delta_B = A\sin(\Omega\tau + \arcsin z/A)\exp(-\gamma_f\tau/2)$, where $A = [(z^2 + y^2 - yz\gamma_f/2H)/(1 - \gamma_f^2/16H^2)]^{1/2}$, while $z = \rho_{11}(\tau_A) - 1/2$ and $y = \mathrm{Im}\rho_{12}(\tau_A + 0) = \mathrm{Re}\rho_{12}(\tau_A - 0)$ are given by Eqs. (3) and (4). This expression considerably simplifies for weak dephasing, $\gamma_A\tau_A \ll 1$ and $\gamma_f \ll H$, when

$$\delta_B = \frac{1}{2}\sin[\Omega\tau + \arcsin(2\rho_{11}(\tau_A) - 1)]\exp(-\frac{\gamma_f\tau}{2}). \quad (6)$$

In contrast to Eq. (5), now the phase of oscillations of $\delta_B(\tau)$ depends on the result Q_A of the first measurement, while the amplitude is maximum possible and independent of Q_A. This fact is very important since it *proves* that after the first measurement (by an ideal detector) the qubit remains in the pure state for *any* result Q_A. This state depends on Q_A and is not one of the localized states as somebody could naively expect.

In a realistic experimental situation the assumption of strong coupling with detectors may be inapplicable. In this case the full probability distribution $P(Q_A, Q_B|\tau)$ as well as the dependence $\overline{Q}_B(Q_A, \tau)$ should be calculated numerically using Eqs. (1)–(2). The results of these calculations for $(\Delta I_A)^2/HS_A = (\Delta I_B)^2/HS_B = 1$ are shown in Figs. 2c and 2d. Weak coupling as well as the nonideality of the detectors decrease the correlation between the results of two measurements, however, for moderate values of the coupling and nonideality the correlation is still significant.

Experimental demonstration of the correlation and agreement with the results of the Bayesian formalism would prove the validity of this formalism and therefore confirm its

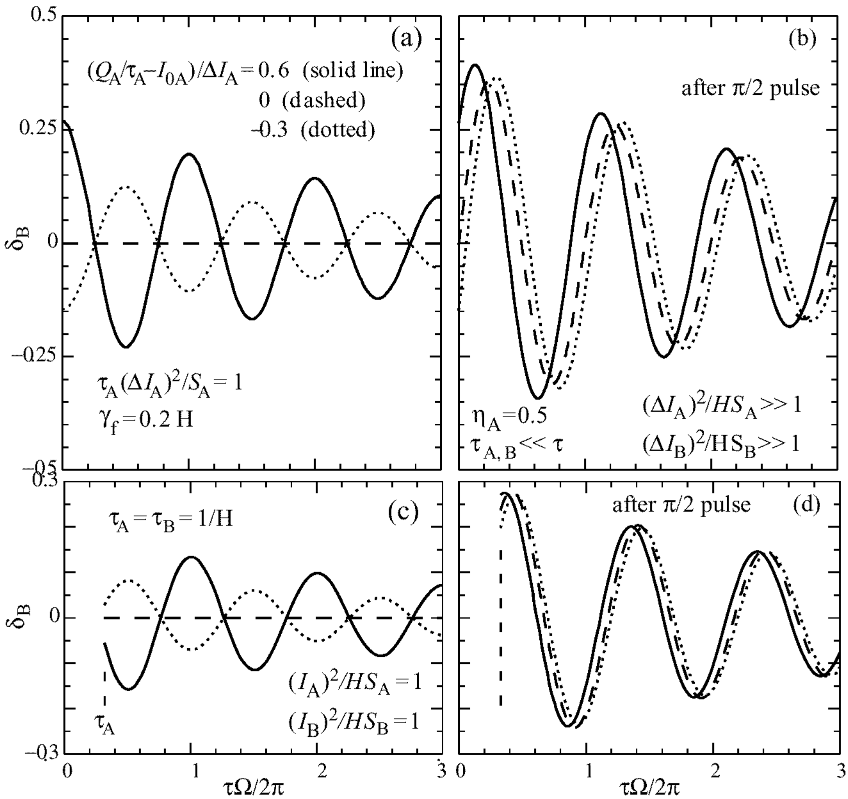

Fig. 2. The normalized average result δ_B of the second measurement for several selected results Q_A of the first measurement, as a function of the time τ between measurements. Panels (a)–(b) are for strong coupling and panels (c)–(d) for moderate coupling between the qubit and detectors.

other predictions. In particular, it would open the way to the qubit purification using continuous quantum feedback control.

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